Ways to Change Carrier Concentrations

Current Flow: Motion of electron and hole

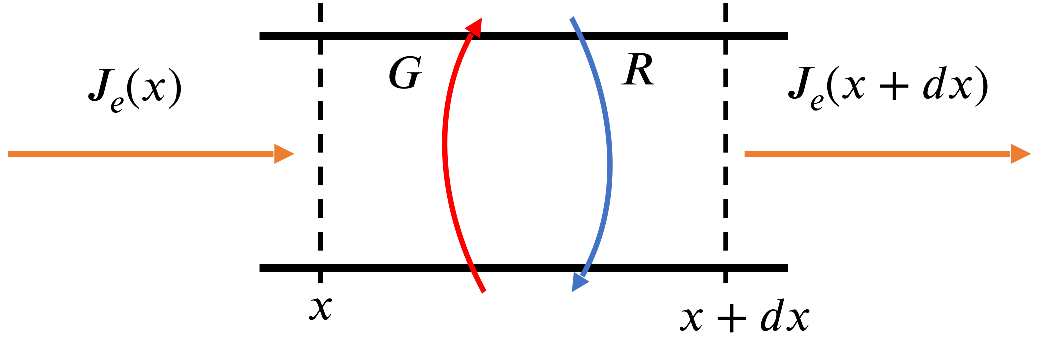
Generation: Local creation of electron-hole pairs

Recombination: Local annihilation of electron-hole pairs

Non-equilibrium Condition

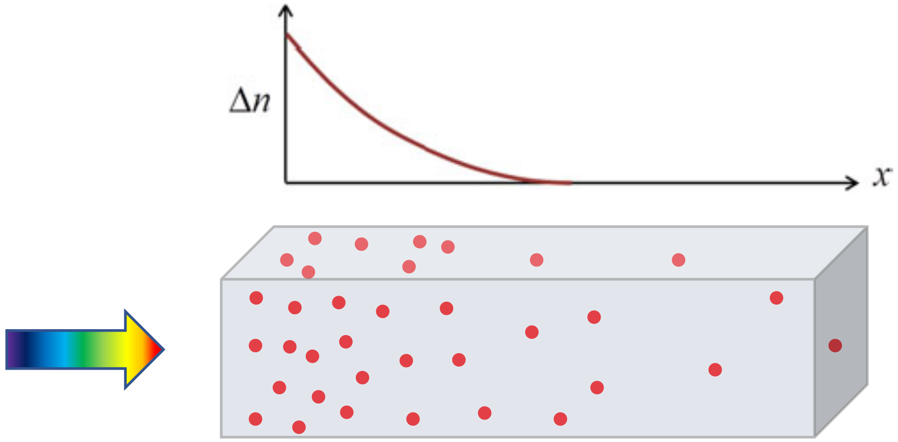
Non-equilibrium Condition

The combined effect of all types of carrier action within a semiconductor, drift, diffusion, and recombination-generation is described by the following continuity equation.



$$\frac{dn}{dt}Adx = \left(\frac{J_{e(total)}(x)}{-e} - \frac{J_{e(total)}(x + dx\ )}{-e}\right)A + (G_e - R_e)Adx$$

G_e is the generation rate
R_e is the recombination rate



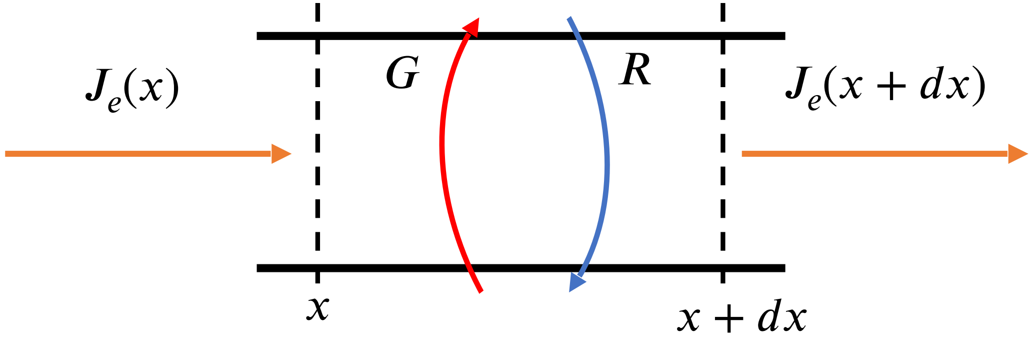
- For the sake of simplicity, assume that all light is absorbed at the surface: electron generation at the surface only. ($\boldsymbol{G_{opt}}$, x > 0) = 0
- No electric field: $\boldsymbol{J_{e(drift)}}$= 0

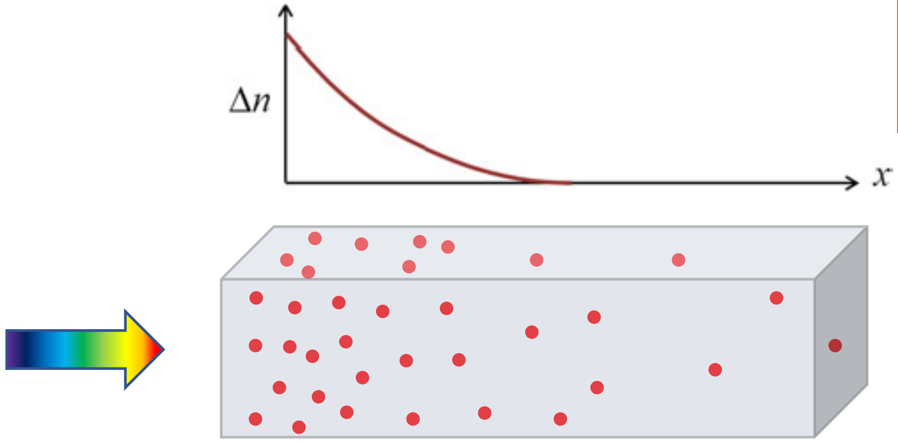
$$\boldsymbol{\Delta n(x) = \Delta n(0) exp\left(-\frac{x}{\sqrt{D_e \tau_e}}\right) = \Delta n(0) exp\left(-\frac{x}{L_e}\right)}$$

$\boldsymbol{\tau_e}$ = carrier lifetime $\qquad$ $\boldsymbol{L_e = \sqrt{D_e \tau_e}}$ **(diffusion length)**

Non-equilibrium Condition

The combined effect of all types of carrier action within a semiconductor, drift, diffusion, and recombination-generation is described by the following continuity equation.





- **The excess electron density decreases exponentially as the distance from the light illuminated front surface increases and the carrier diffusion length decreases.**

- No electric field: $J_{e(drift)} = 0$

$$\Delta n(x) = \Delta n(0) exp\left(-\frac{x}{\sqrt{D_e \tau_e}}\right) = \Delta n(0) exp\left(-\frac{x}{L_e}\right)$$

$$L_e = \sqrt{D_e \tau_e} \text{ (diffusion length)}$$

Non-equilibrium Carrier Concentration

Under non-equilibrium conditions, an excess or deficiency of charge carriers exists in a semiconductor (under illumination).

- An increase in the densities of the excess carriers is accompanied by a shift of the quasi-Fermi levels.
- As the electron density increases, the quasi-Fermi level of the electron steadily approaches the CB.
- In contrast, the quasi-Fermi level of the hole approaches the VB for increasing hole densities.
- In this way, the quasi-Fermi levels of electron and hole shift in opposite directions, and their energy difference becomes larger as the carrier densities increase.

$$n = n_0 + \Delta n$$

$$p = p_0 + \Delta p$$

n and p: The electron and the hole density under non-equilibrium.

n_0 and p_0: The electron and the hole density under equilibrium.

$$n = N_C \, exp\left[\frac{(E_{Fn} - E_C)}{k_B T}\right] = n_i \, exp\left[\frac{(E_{Fn} - E_i)}{k_B T}\right]$$

$$p = N_V \, exp\left[\frac{(E_V - E_{Fp})}{k_B T}\right] = n_i \, exp\left[\frac{(E_i - E_{Fp})}{k_B T}\right]$$

E_{Fn}: The quasi-Fermi energy for electrons.
E_{Fp}: The quasi-Fermi energy for holes.

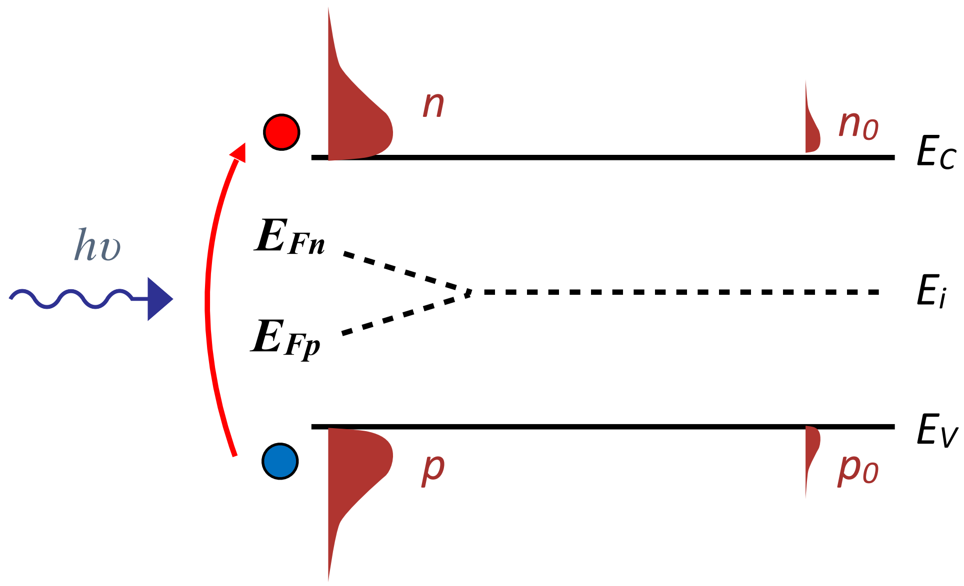
→

$$E_{Fn} = E_i + k_B T ln(\frac{n}{n_i})$$

$$E_{Fp} = E_i - k_B T ln(\frac{p}{n_i})$$

Non-equilibrium Carrier Concentration

Under non-equilibrium conditions, an excess or deficiency of charge carriers exists in a semiconductor (under illumination).



$$n = n_0 + \Delta n$$

$$p = p_0 + \Delta p$$

$\boldsymbol{n}$ and $\boldsymbol{p}$: The electron and the hole density under non-equilibrium.

$\boldsymbol{n_0}$ and $\boldsymbol{p_0}$: The electron and the hole density under equilibrium.

$$n = N_C\,exp\left[\frac{(E_{Fn} - E_C)}{k_BT}\right] = n_i\,exp\left[\frac{(E_{Fn} - E_i)}{k_BT}\right]$$

$$p = N_V\,exp\left[\frac{(E_V - E_{Fp})}{k_BT}\right] = n_i\,exp\left[\frac{(E_i - E_{Fp})}{k_BT}\right]$$

E_{Fn}: The quasi-Fermi energy for electrons.

E_{Fp}: The quasi-Fermi energy for holes.

→

$$E_{Fn} = E_i + k_BTln(\frac{n}{n_i})$$

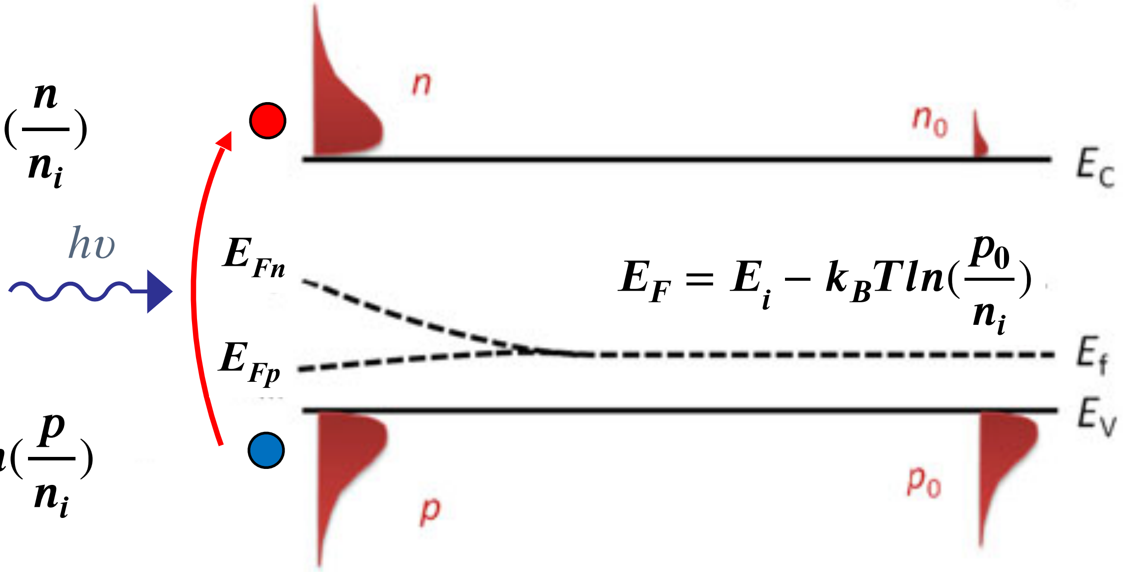
$$E_{Fp} = E_i - k_BTln(\frac{p}{n_i})$$

Non-equilibrium Carrier Concentration

- In case of a **p-type SC**, $p_0 >> n_0$.
- The excess electrons form the electron quasi-Fermi level.
- The contribution of the excess carriers to the total carrier density is **smaller for the holes**.
- The shift in the hole quasi-Fermi level is also smaller.

$$n = n_0 + \Delta n \approx \Delta n \qquad E_{Fn} = E_i + k_B T ln\left(\frac{n}{n_i}\right)$$

$$p = p_0 + \Delta p \approx p_0 \qquad E_{Fp} = E_i - k_B T ln\left(\frac{p}{n_i}\right)$$



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Non-equilibrium Carrier Concentration

Example) A piece of Si doped with 10^{16} cm^{-3} shallow donors is illuminated with light generating 10^{15} cm^{-3} excess electrons and holes. Calculate the quasi-Fermi energies relative to the intrinsic energy and compare it to the Fermi energy in the absence of illumination.

Solution)

$$n = n_0 + \Delta n = 10^{16} + 10^{15} = 1.1\times 10^{16} cm^{-3}$$

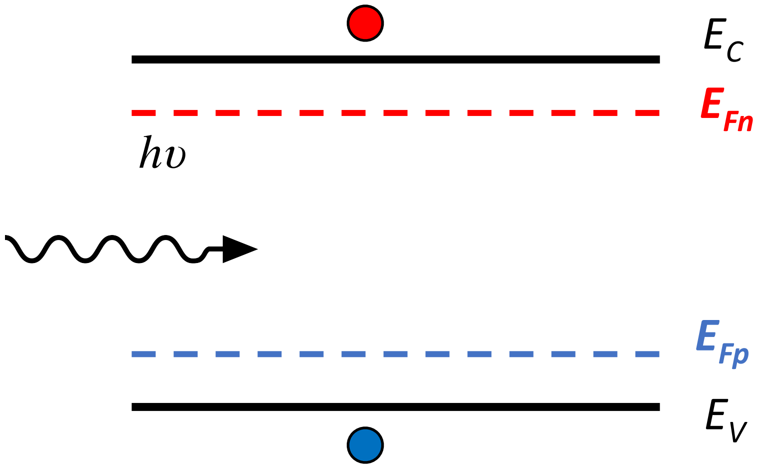
$$p = p_0 + \Delta p = 10^{15} cm^{-3}$$

the number of minority carriers in an illuminated solar cell can be approximated by the number of light generated carriers

The quasi-Fermi levels under illumination:

$$E_{Fn} - E_i = k_B T ln\left(\frac{n}{n_i}\right) = 0.0259 \times \ln\left(\frac{1.1\times 10^{16}}{10^{10}}\right) = 360\ meV$$

$$E_i - E_{Fp} = k_B T ln\left(\frac{p}{n_i}\right) = 0.0259 \times \ln\left(\frac{1\times 10^{15}}{10^{10}}\right) = 298\ meV$$

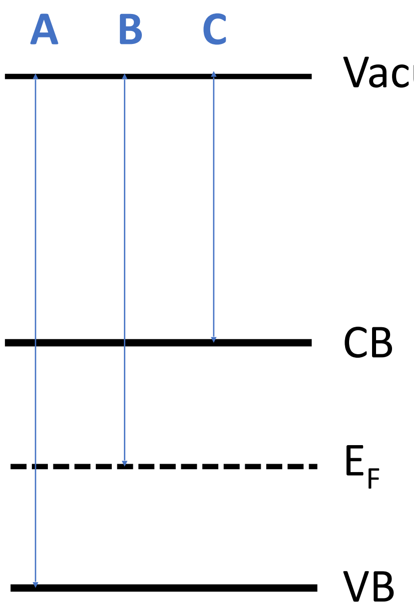


The Fermi levels in absence of light:

$$E_F - E_i = k_B T ln\left(\frac{n}{n_i}\right) = 0.0259 \times \ln\left(\frac{1\times 10^{16}}{10^{10}}\right) = 358\ meV$$

Before moving to Junctions!

Work function (WF) Ionization energy (IE) Electron affinity (EA)



A: Ionization energy (IE)

B: Work function (WF)

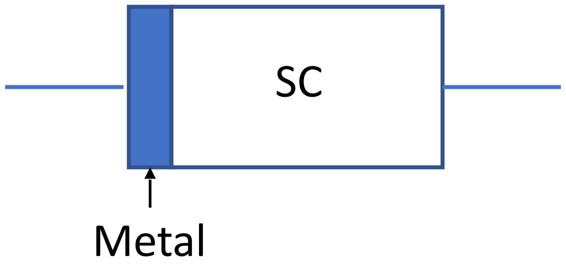
C: Electron affinity (EA)

Ionization Energy is the energy required to remove the outermost electron in an atom or molecule in its ground state to infinity, i.e. cause the atom to become ionized.

Work Function (ϕ) is the amount of energy required to transfer an electron from the E_F to the vacuum level.

Electron affinity (χ) is defined as the potential energy change of an atom or molecule when an electron is added to a neutral gaseous atom to form a negative ion.

Metal-Semiconductor Junction

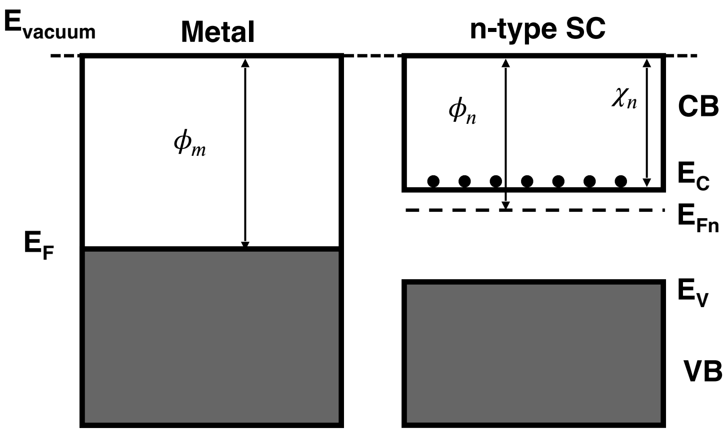


Metal-SC (M-S) junctions can behave as either Schottky barriers or as Ohmic contacts, depending on the interface properties.

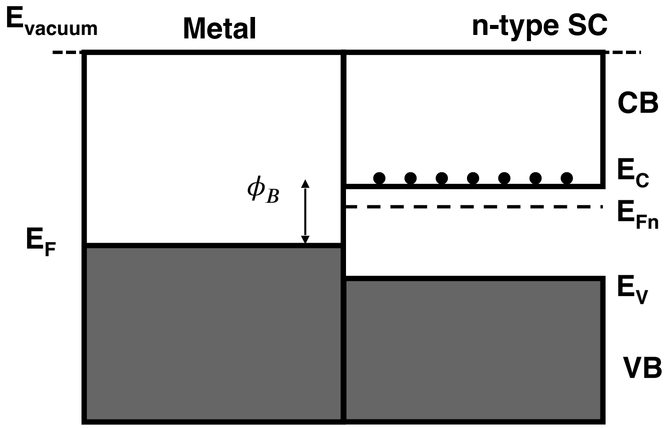
Schottky junction: $\phi_m > \phi_{sc}$

Ohmic junction: $\phi_m < \phi_{sc}$

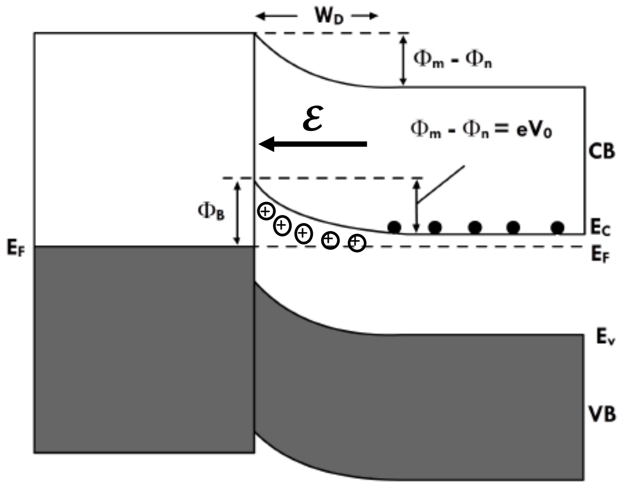
Before contact



After contact



In thermal equilibrium



Ionization E, Electron Affinity etc: https://pubs.rsc.org/en/content/articlehtml/2016/mh/c5mh00160a

W_d: Depletion width

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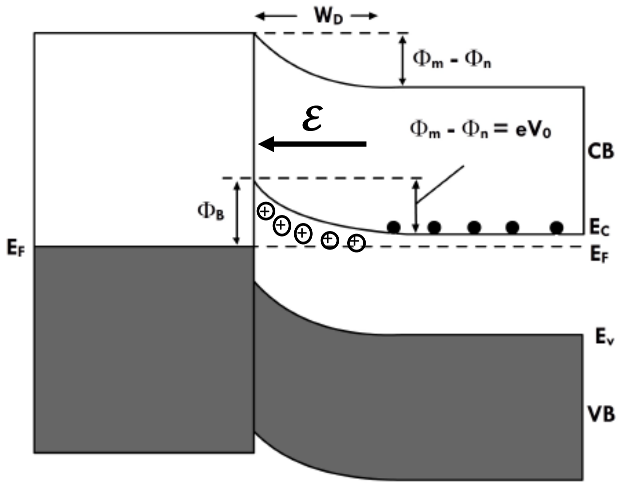
Metal-Semiconductor Junction

- The barrier height, Φ_B, is defined as the potential an electron overcome to cross the junction.
- A built in potential in the Schottky junction, V_0 is a constant potential difference across the junction at equilibrium = difference between the work functions of materials at equilibrium.
- When a Schottky junction is formed between the metal and semiconductor, **a positive potential is formed on the SC side. Because the depletion region extends within a certain depth in the SC, there is bending of the energy bands on the SC side.**
- Bands bend up in the direction of the electric field (field goes from positive charge to negative charge): the energy bands bend up going from n-type SC to metal
- There is a certain region in the SC (denoted by W) where the bands bend (this is the depletion region). Another name for the **depletion region** is the **space charge layer**.

In thermal equilibrium



W_d: Depletion width

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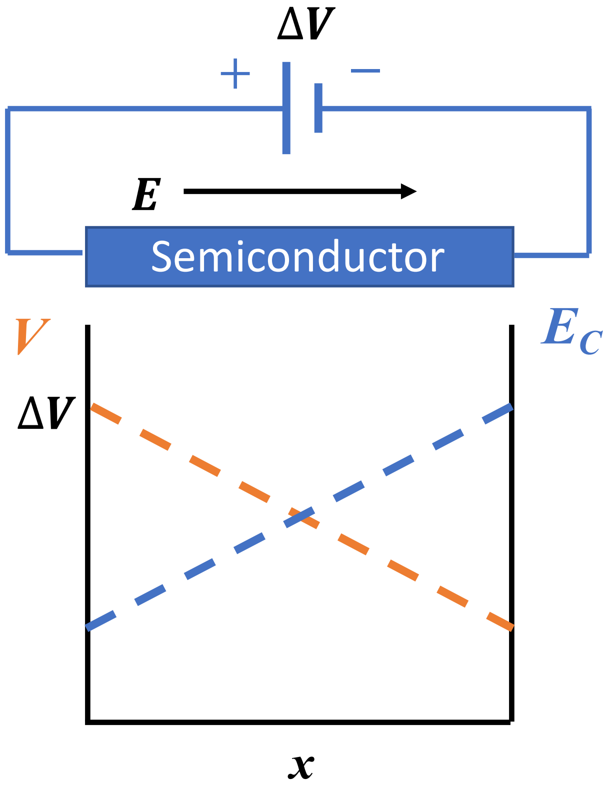
Electric Potential vs Electric Field

- The electric field (E) is related to the electric potential by $E(x) = -\frac{dV}{dx}$
- The difference in potential energy between 2 points in an E-field containing a charge Q: $\Delta PE = Q \cdot E \cdot \Delta x$

$$E(x) = -\frac{dV}{dx} = \frac{1}{Q}\frac{dPE}{dx}$$

PE is E_C or E_V!

- High electric potential (V) correlates with a high concentration of positive charge.



More positive voltage lowers E_C and E_V.

More negative voltage raises E_C and E_V.

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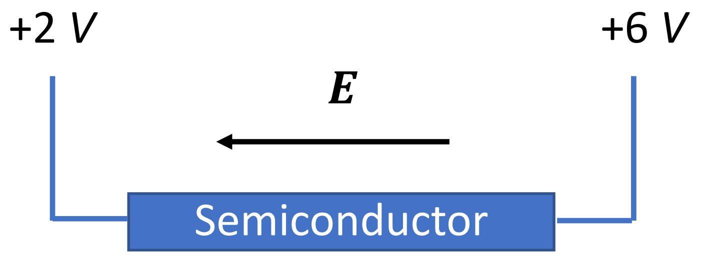
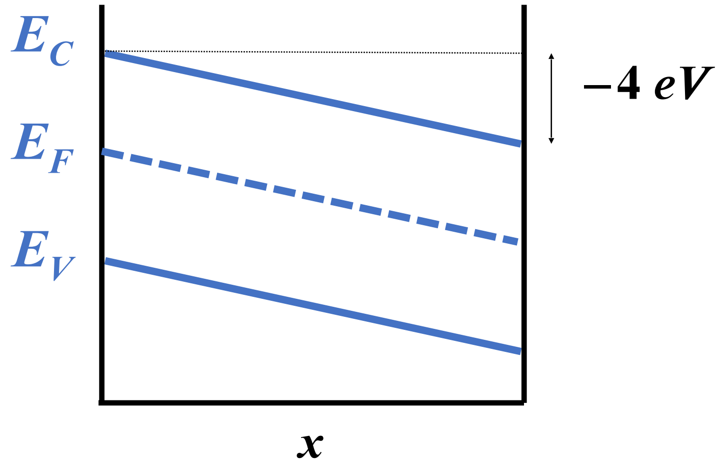
Electric Potential vs Electric Field

Example)



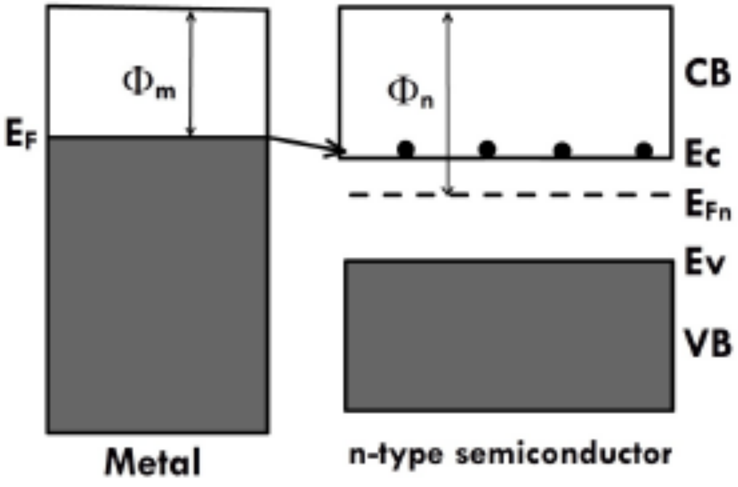
$$E(x) = -\frac{dV}{dx} = \frac{1}{Q}\frac{dPE}{dx}$$

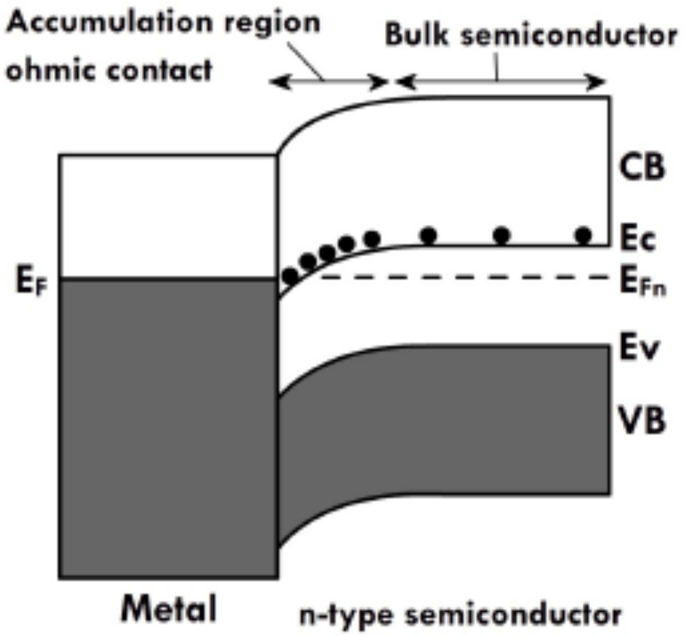
$$\Delta PE = Q \cdot \Delta V = -e \cdot (4V) = -4\ eV$$



- The Fermi level is also slanted with E_C and E_V because there is current flow.
- The slant is opposite for a hole but the energy diagram shows in general electron energy.

Metal-Semiconductor Ohmic Junction

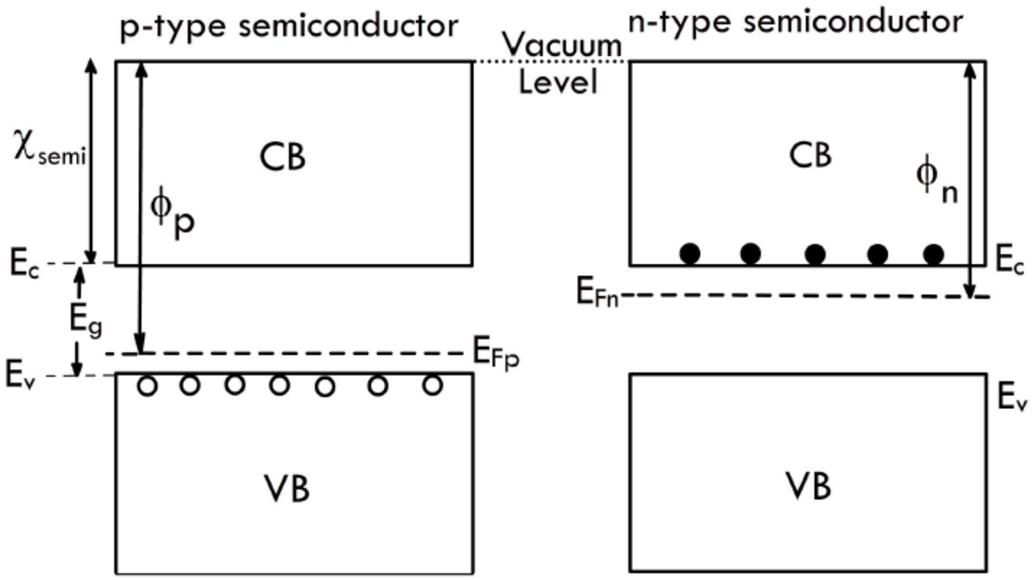




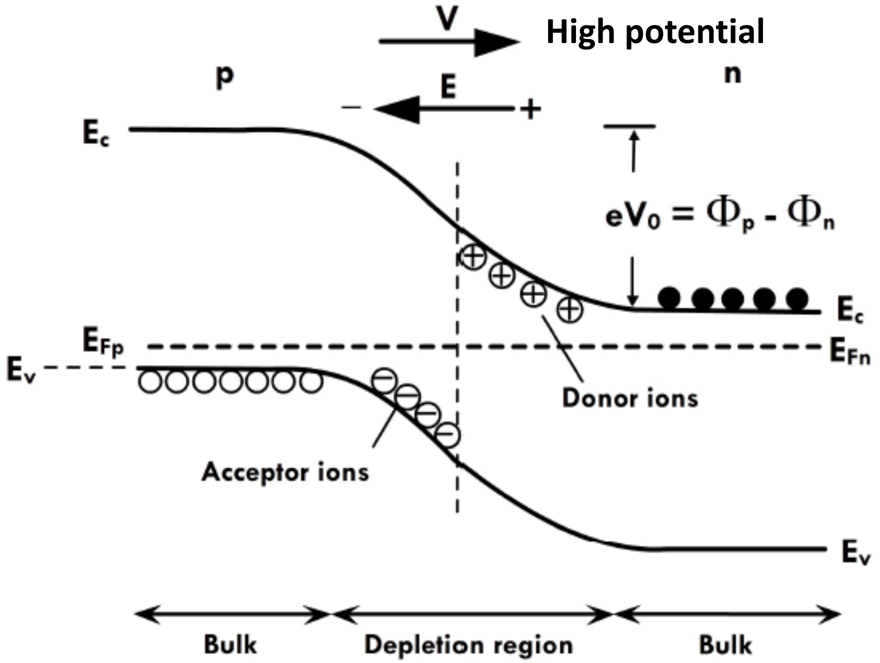
pn Junction (Homojunction or Heterojunction)

- In thermal equilibrium after contact, the Fermi levels line up at equilibrium.
- There are excess electrons on the n-side and excess holes on the p-side.
- The electrons move from the n side to the p side and combine with the holes, while holes move from p to n side and combine with the electrons.
- Because of the movement of majority carriers a net positive charge develops on the n-side (due to the positively charged donor ions) and a negative charge develops on the p-side (due to negatively charged acceptor ions).
- Thus, an E-field is formed going from the n-side to the p-side and there is a built-in potential, from the p to the n side.

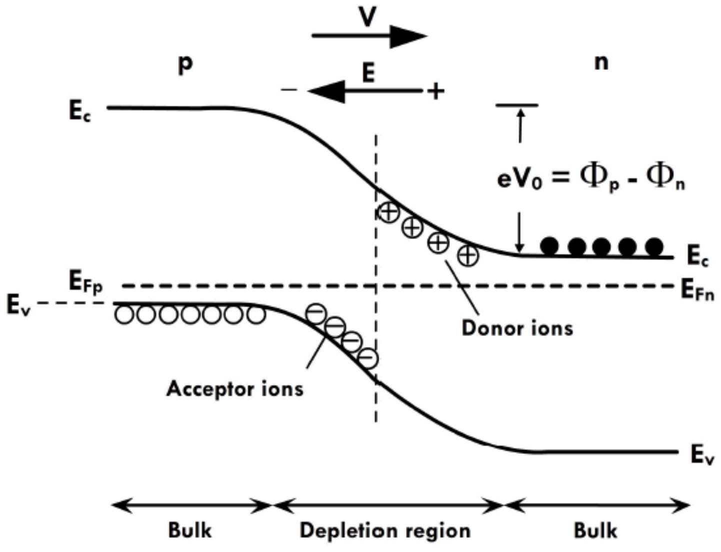
Before contact



In thermal equilibrium



Depletion Region in pn Junction

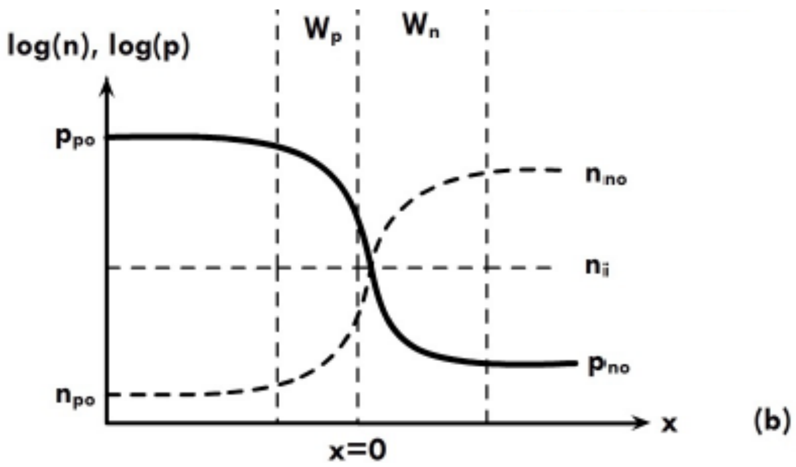


The total depletion region width: $w_0 = w_p + w_n$

Charge balance in the region: $N_A A w_p = N_D A w_n$

The ratio of the depletion widths: $\dfrac{w_p}{w_n} = \dfrac{N_D}{N_A}$

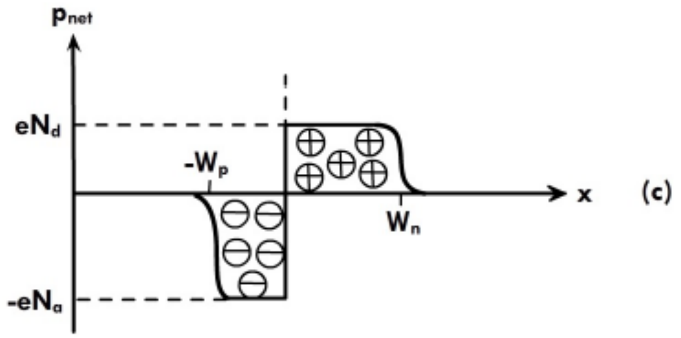
$w_p < w_n \ if \ N_A > N_D$ $\quad$ $w_p \ll w_n \ if \ N_A \gg N_D$ $\quad$ p$^+$n junction

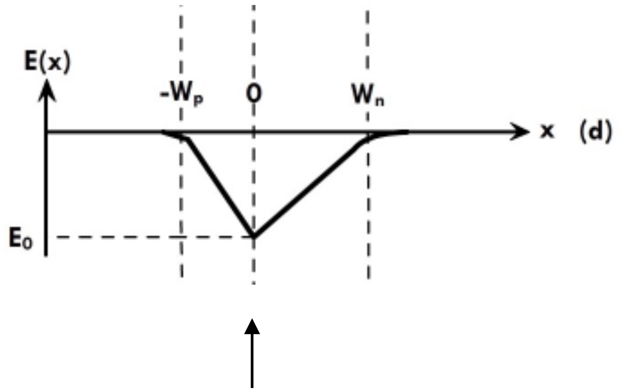


Carrier concentrations in the p side: $p_{p0} = N_A$ $\quad$ $n_{p0} = \dfrac{n_i^2}{N_A}$

Carrier concentrations in the n side: $n_{n0} = N_D$ $\quad$ $p_{n0} = \dfrac{n_i^2}{N_D}$

Depletion Region in pn Junction





At x = 0 the E-field must match:

$$E(0) = -\frac{eN_A w_p}{\varepsilon} = -\frac{eN_D w_n}{\varepsilon}$$

The charge density in the depletion region = abrupt charge distribution:

$$\rho_{net} = -eN_A \qquad for\ -w_p < x < 0$$

$$\rho_{net} = eN_D \qquad for\ 0 < x < w_n$$

For 1 dimensional interface by Gauss law: $\frac{dE}{dx} = \frac{\rho_{net}}{\varepsilon}$

The field obtained by integrating the total charge density:

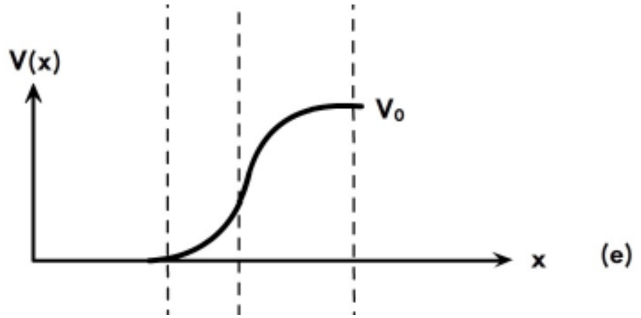
$$E(x) = \frac{1}{\varepsilon}\int \rho_{net}(x)dx$$

With the boundary condition that the field must go to zero at the boundaries i.e. x = -w_p and x = w_n:

$$E(x) = -\frac{eN_A}{\varepsilon}(x + w_p) \qquad for\ -w_p < x < 0$$

$$E(x) = \frac{eN_D}{\varepsilon}(x - w_n) \qquad for\ 0 < x < w_n$$

Depletion Region in pn Junction



The electric field is related to the potential by $E(x) = -\frac{dV}{dx}$

With the boundary conditions are set such that V = 0 at x = $-w_p$ and V = V_0 at x = w_n, where V_0 is the total potential i.e. built-in potential:

$$V(x) = \frac{eN_A}{2\varepsilon}(x + w_p)^2 \qquad for\ -w_p < x < 0$$

$$V(x) = V_0 - \frac{eN_D}{2\varepsilon}(x - w_n)^2 \qquad for\ 0 < x < w_n$$

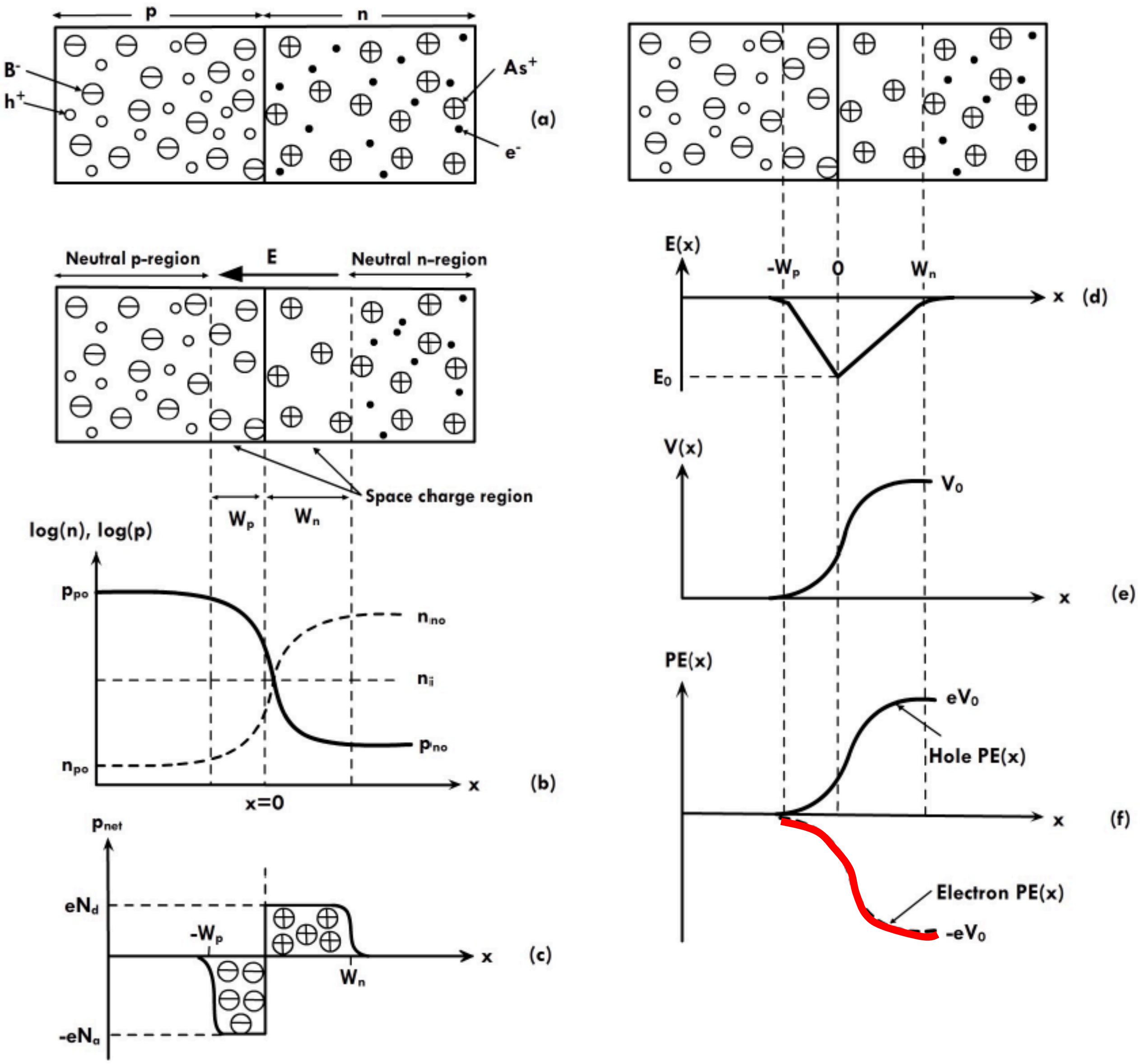
The potential function should be continuous and V should be equal at x = 0: $\frac{eN_A}{2\varepsilon}{w_p}^2 = V_0 - \frac{eN_D}{2\varepsilon}{w_n}^2$

Rearrangement and using $\frac{w_p}{w_n} = \frac{N_D}{N_A}$ $and\ w_0 = w_p + w_n$: $w_0 = \sqrt{\frac{2\varepsilon V_0}{e}\left(\frac{N_A + N_D}{N_A N_D}\right)}$

Th built-in potential: $V_0 = E_{Fn} - E_{Fp} = k_B T ln\left(\frac{np}{n_i^2}\right)$

from $E_{Fn} - E_i = k_B T ln\left(\frac{n}{n_i}\right)$ and $E_i - E_{Fp} = k_B T ln\left(\frac{p}{n_i}\right)$

Depletion Region in pn Junction



(a) Schematic of the pn junction
(b) Carrier concentration across the junction
(c) Charge density variation
(d) Electric field
(e) Potential
(f) Potential energy in the depletion region

Depletion Region in pn Junction

Example) An abrupt silicon p-n junction consists of a p-type region containing 5 x 10^{16} cm^{-3} acceptors and an n-type region containing 10^{17} cm^{-3} donors.

a. Calculate the thermal equilibrium density of electrons and holes in the p-type region as well as both densities in the n-type region.

b. Calculate the built-in potential of the p-n junction.

c. Calculate the built-in potential of the p-n junction at 400 K.

Solution) a. In the p-type region: $p = N_A = 5 \times 10^{16}\ cm^{-3}$ $\quad n = \frac{n_i^2}{p} = \frac{10^{20}}{5 \times 10^{16}} = 2 \times 10^3\ cm^{-3}$

In the n-type region: $n = N_D = 10^{17}\ cm^{-3}$ $\quad p = \frac{n_i^2}{n} = \frac{10^{20}}{10^{17}} = 1 \times 10^3\ cm^{-3}$

b. The built-in potential: $E_{Fn} - E_i = k_B T ln\left(\frac{n}{n_i}\right)$ $\quad E_i - E_{Fp} = k_B T ln\left(\frac{p}{n_i}\right)$

$$\boldsymbol{E_{Fn} - E_{Fp} = k_B T ln\left(\frac{np}{n_i^2}\right)} = 0.0259 \times \ln\left(\frac{10^{17} \times 5 \times 10^{16}}{10^{20}}\right) = 0.817\ V$$

c. The built-in potential at 400 K:

$$\boldsymbol{E_{Fn} - E_{Fp} = k_B T ln\left(\frac{np}{n_i^2}\right)} = 0.0345 \times \ln\left(\frac{10^{17} \times 5 \times 10^{16}}{(4.52 \times 10^{12})^2}\right) = 0.500\ V$$

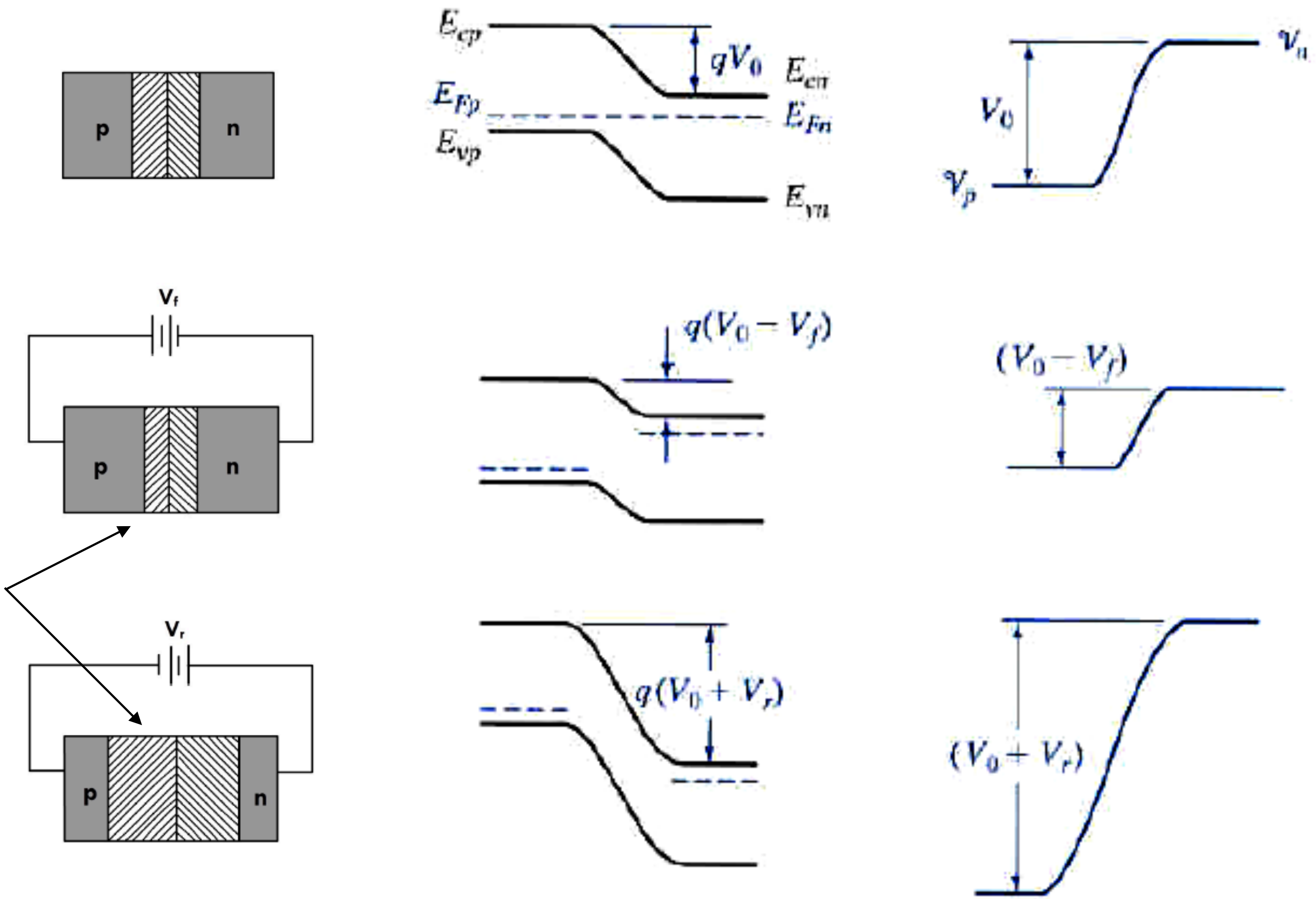
pn Junction Under Bias

Energy band | **Potential barrier**

Equilibrium

Forward bias

Reverse bias



$$w_0 = \sqrt{\frac{2\varepsilon(V_0 - V_{ext})}{e}\left(\frac{N_A + N_D}{N_A N_D}\right)}$$

pn Junction Under Bias

Example) An abrupt silicon (n_i = 10^{10} cm^{-3}) p-n junction consists of a p-type region containing 5 x 10^{16} cm^{-3} acceptors and an n-type region containing 10^{17} cm^{-3} donors.

a. Calculate the total width of the depletion region if the applied voltage equals 0, 0.5 and -2.5 V.

b. Calculate the potential across the depletion region in the n-type semiconductor at 0, 0.5 and -2.5 V.

c. Calculate maximum electric field in the depletion region at 0, 0.5 and -2.5 V.

Solution) The built-in potential of this p-n junction was calculated and the value was 0.814 V.

a. Calculate the total width of the depletion region: $w_0 = \sqrt{\frac{2\varepsilon(V_0 - V_{ext})}{e}\left(\frac{N_A + N_D}{N_A N_D}\right)}$

b. Calculate the potential across the depletion region in the n-type semiconductor: $\frac{eN_D}{2\varepsilon}{w_n}^2, \quad w_n = \frac{w_0}{1 + \frac{N_D}{N_A}}$

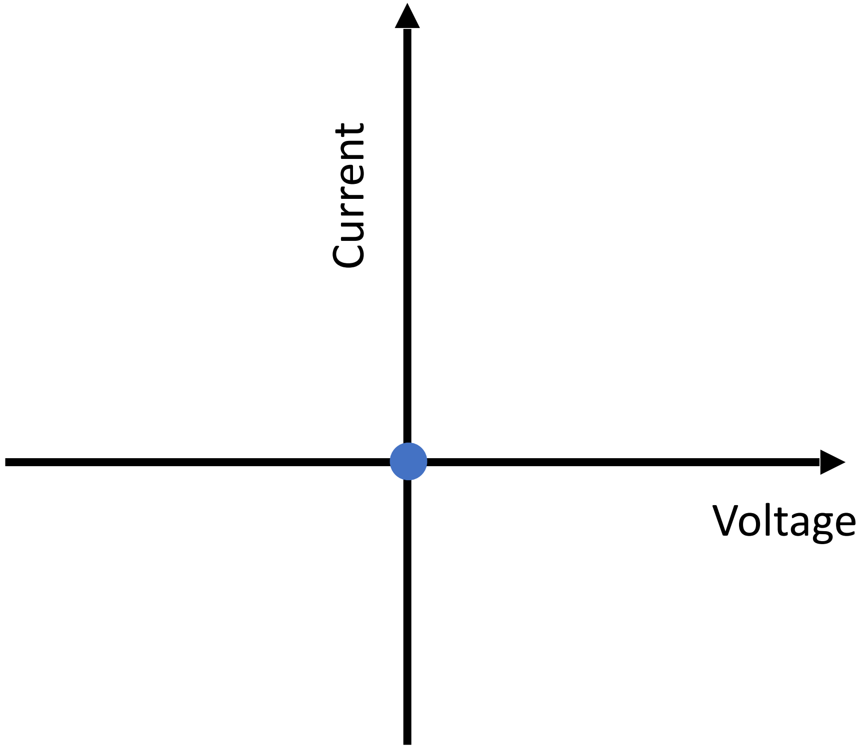
c. Calculate maximum electric field in the depletion region: $E(0) = -\frac{eN_D w_n}{\varepsilon}$

	V_{ext} = 0 V	V_{ext} = 0.5 V	V_{ext} = -2.5 V
w_0 (μm)	0.178	0.111	0.358
V_n (V)	0.272	0.106	1.106
E(0) (kV/cm)	91.9	57.2	185

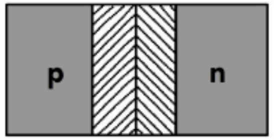
pn Junction Under Bias

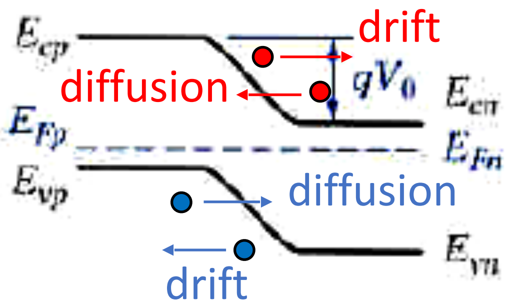
Energy band

Equilibrium

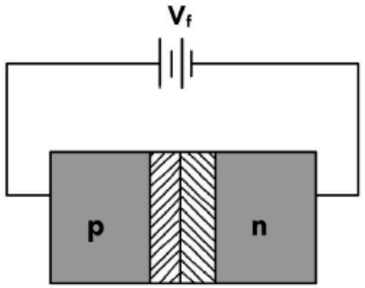


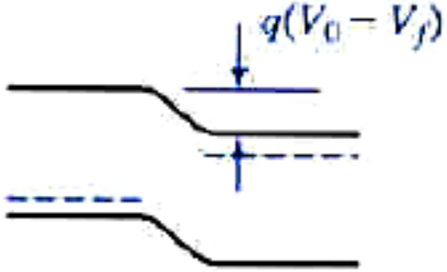
Equilibrium



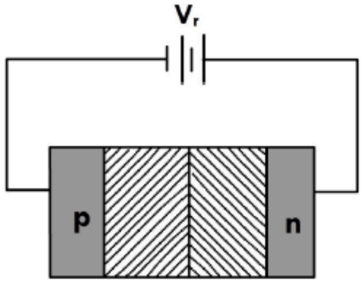


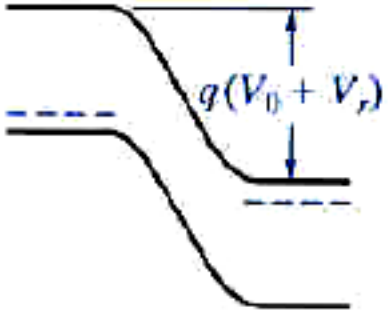
Forward bias





Reverse bias



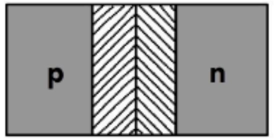


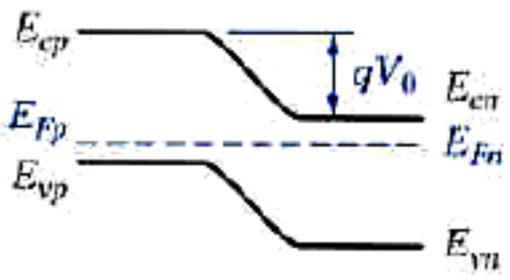
pn Junction Under Bias

Energy band

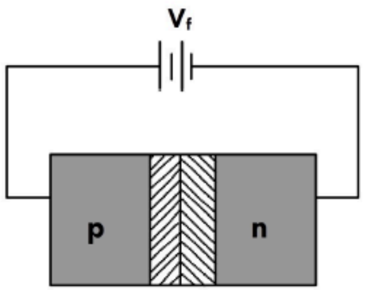
Forward bias

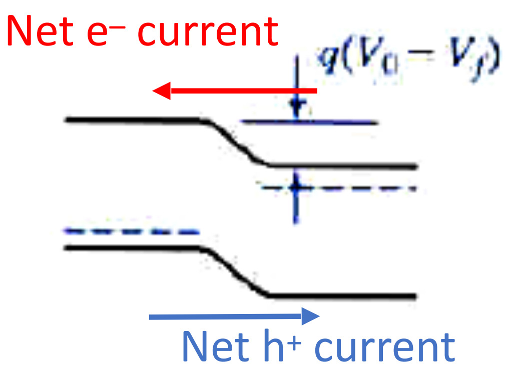
Equilibrium



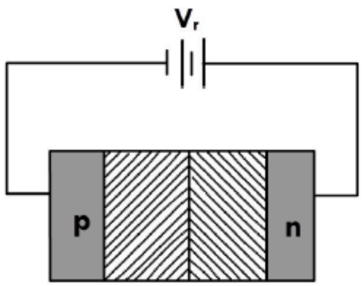


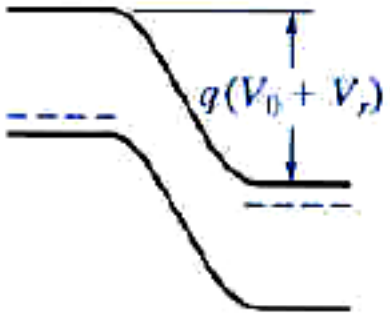
Forward bias

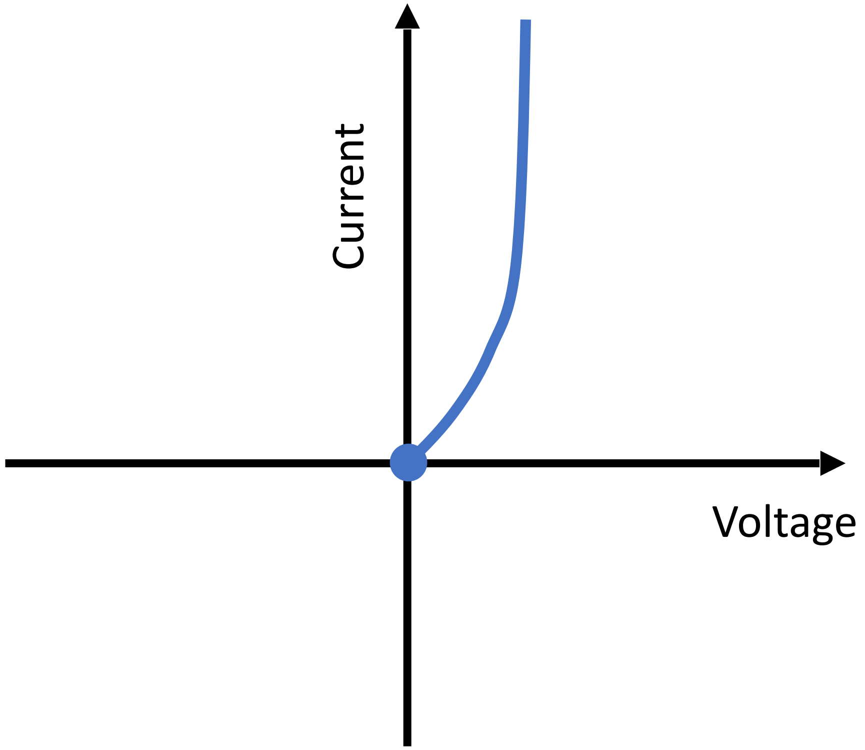




Reverse bias







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pn Junction Under Bias

Energy band

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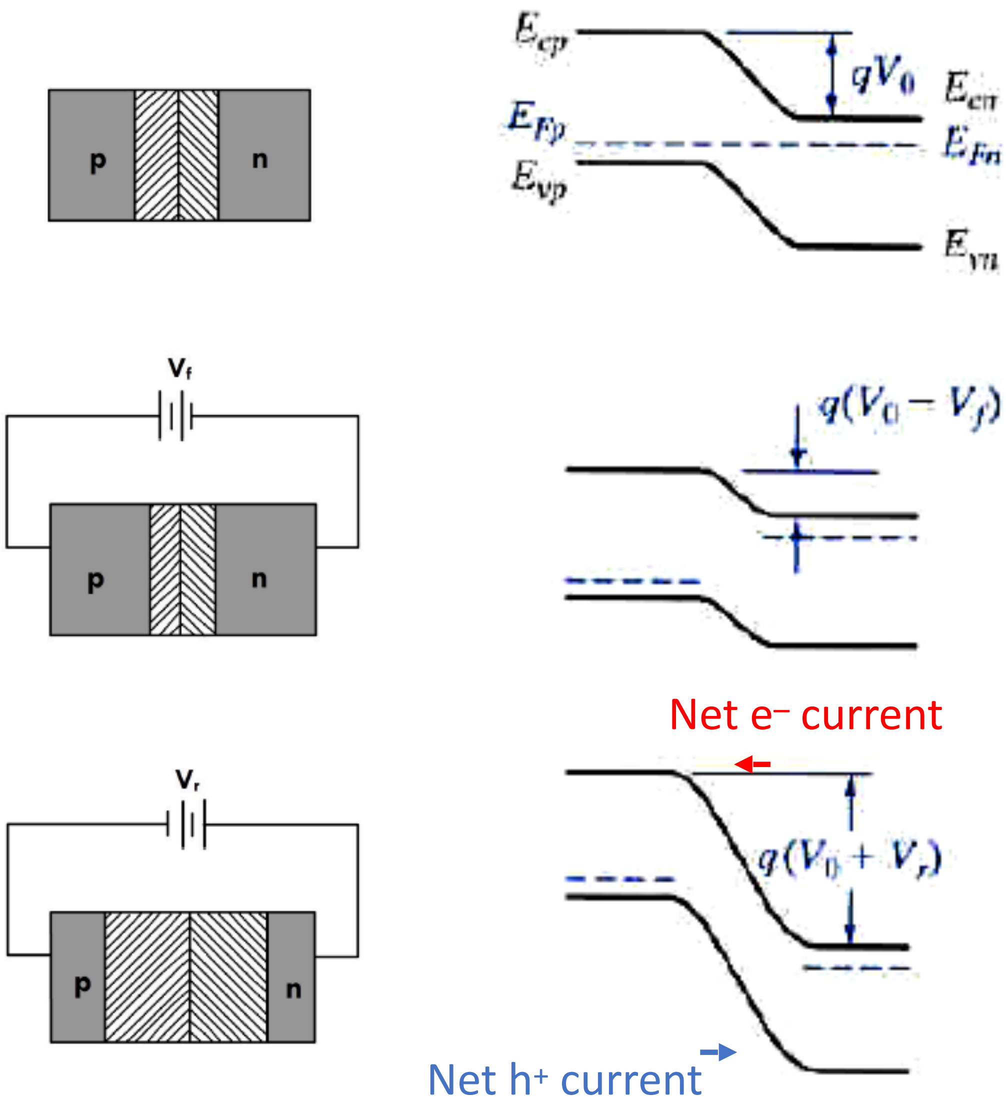
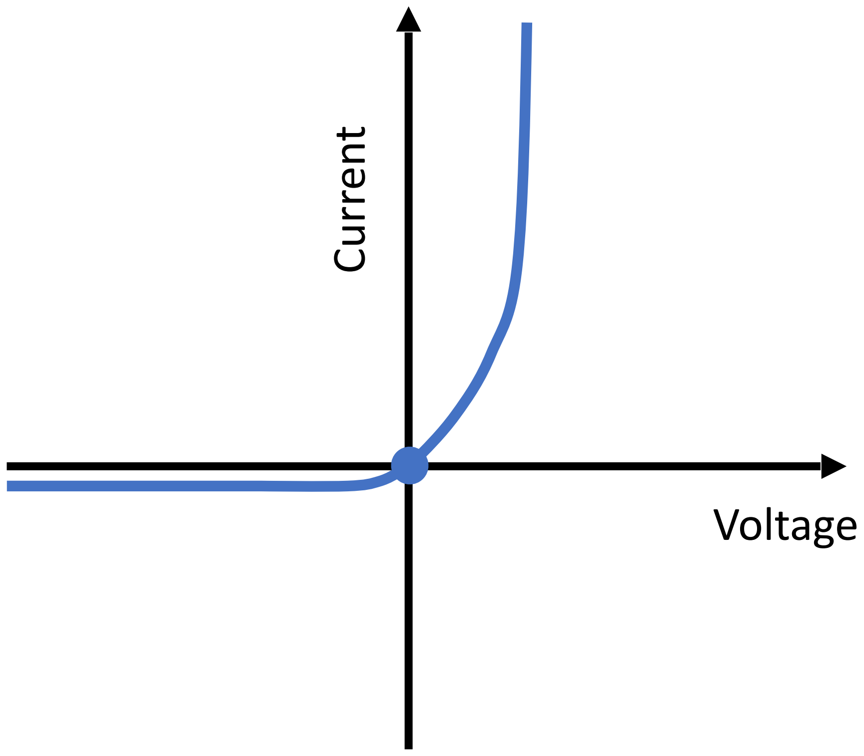
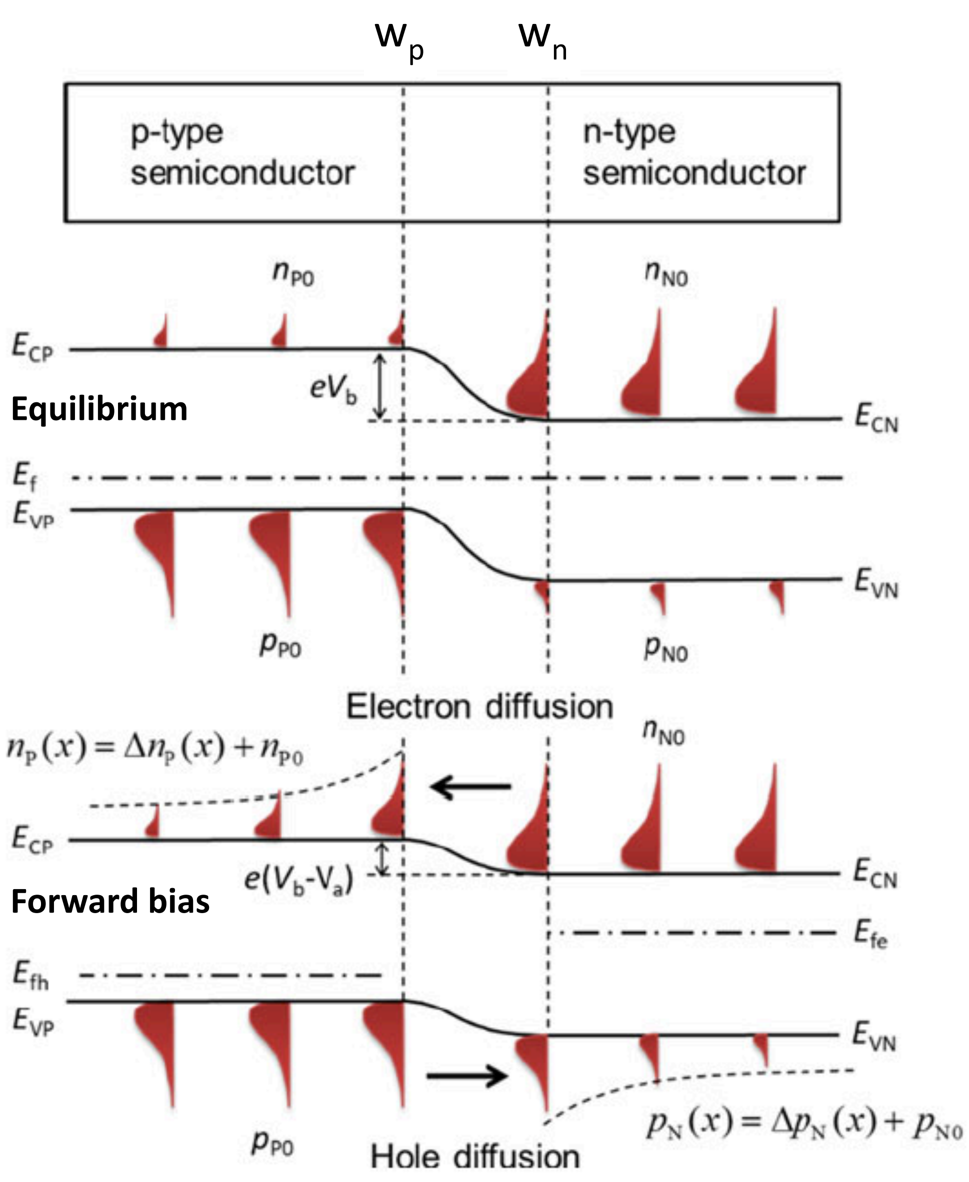


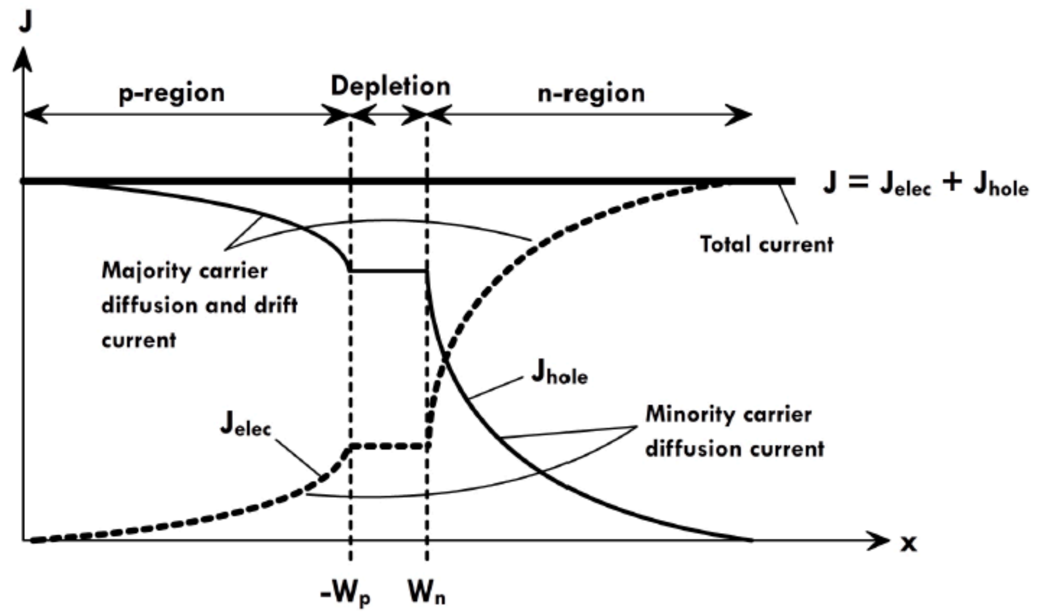
Reverse bias



IV Characteristics of a pn Junction (Forward Bias)



- **Under forward bias**, the potential barrier at the junction interface is lowered.
- The electrons flow from the n-type region into the p-type region while holes flow in the reverse direction.
- The total current is a constant independent of position.
- The dominant carrier transport in a p–n junction is diffusion and the drift current of the minority carriers is not considered.



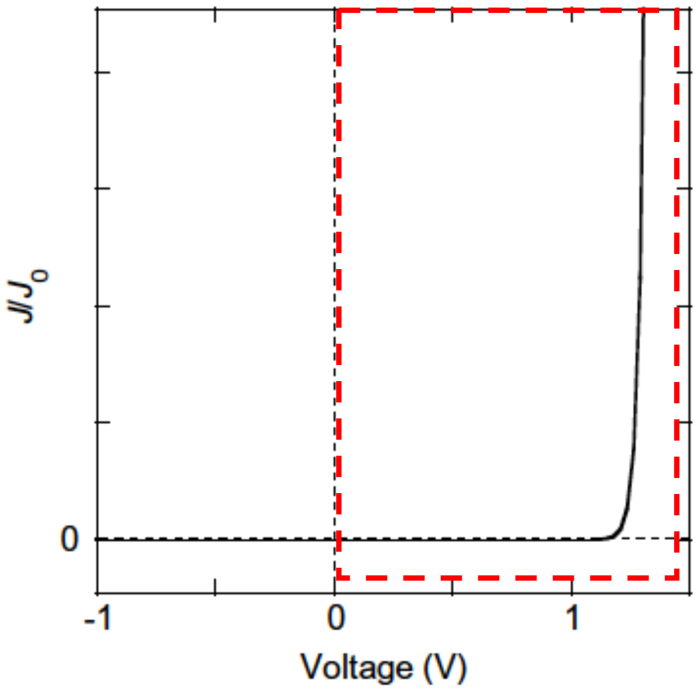
IV Characteristics of a pn Junction (Forward Bias)

$$J_{total} = J_{e(diff)}(w_P) + J_{h(diff)}(w_N)$$

$$= e\left(\frac{D_e n_{P0}}{L_e} + \frac{D_h p_{N0}}{L_h}\right)\left\{exp\left(\frac{eV_a}{k_B T}\right) - 1\right\}$$

$$= J_0\left\{exp\left(\frac{eV_a}{k_B T}\right) - 1\right\}$$

The Shockley diode equation



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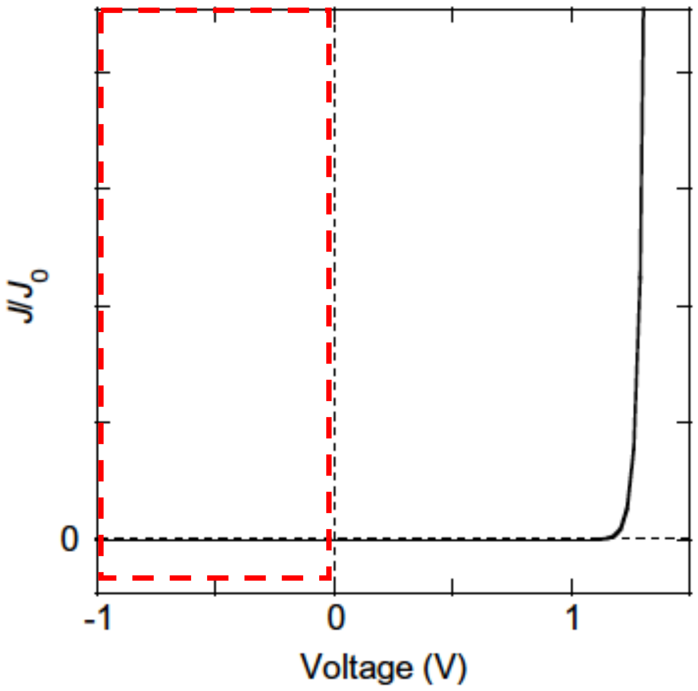
IV Characteristics of a pn Junction (Reverse Bias)

$$J_{total} = J_{e(diff)}(w_P) + J_{h(diff)}(w_N)$$

$$= e\left(\frac{D_e n_{P0}}{L_e} + \frac{D_h p_{N0}}{L_h}\right)\left\{exp\left(\frac{eV_a}{k_B T}\right) - 1\right\}$$

$$= J_0\left\{exp\left(\frac{eV_a}{k_B T}\right) - 1\right\}$$

The Shockley diode equation



$$J_0 = e\left(\frac{D_e n_{P0}}{L_e} + \frac{D_h p_{N0}}{L_h}\right) = e n_i^2\left(\sqrt{\frac{D_e}{\tau_e}}\frac{1}{N_A} + \sqrt{\frac{D_h}{\tau_h}}\frac{1}{N_D}\right)$$

$$= e\left(\sqrt{\frac{D_e}{\tau_e}}\frac{1}{N_A} + \sqrt{\frac{D_h}{\tau_h}}\frac{1}{N_D}\right) N_C N_V exp\left(-\frac{E_g}{k_B T}\right)$$

The saturation current

- The reverse bias causes the potential barrier increases and **the carrier transport across the junction will be inhibited**.
- Under ideal condition, a constant value of J_0 is maintained for wide range of reverse bias voltages.
- If an excessively high reverse bias is applied, a tunnel current flow at the depletion layer leads to a significant increase of the reverse current (at breakdown voltage).

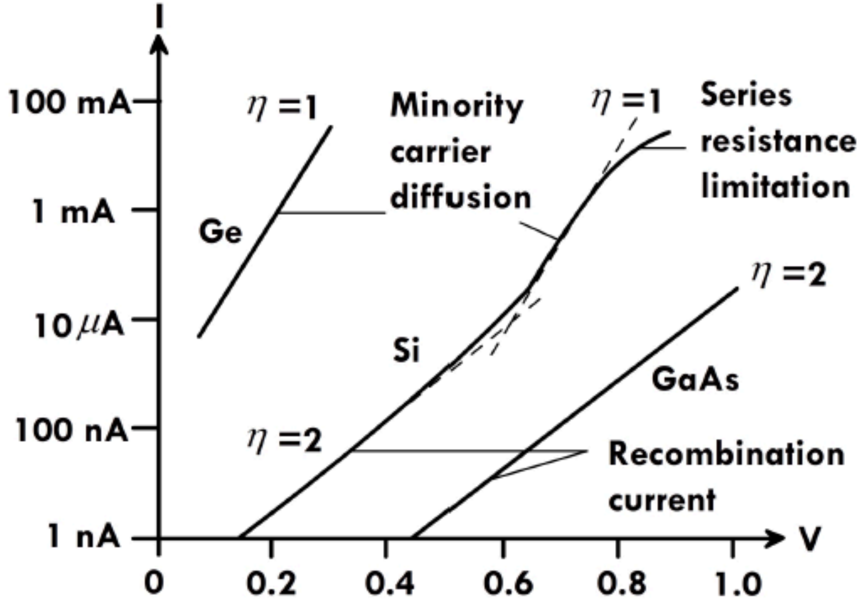
Diode Ideality Factor & Band Gap Dependence

For actual diodes, the equation becomes:

$$J_{total} = J_0\left\{exp\left(\frac{eV_a}{nk_BT}\right) - 1\right\}$$

n = ideality factor, a number between 1 and 2.

The n is close to 1 for the case of an "ideal" like diode and when n shifts to 2, it account for imperfect junctions (mainly due to recombination).

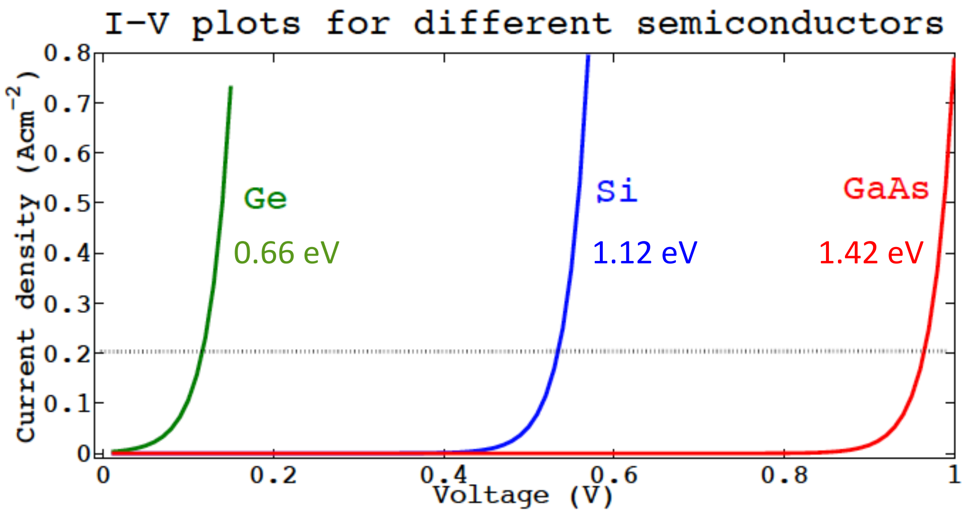


Adapted from Principles of Electronic Materials - S.O. Kasap.

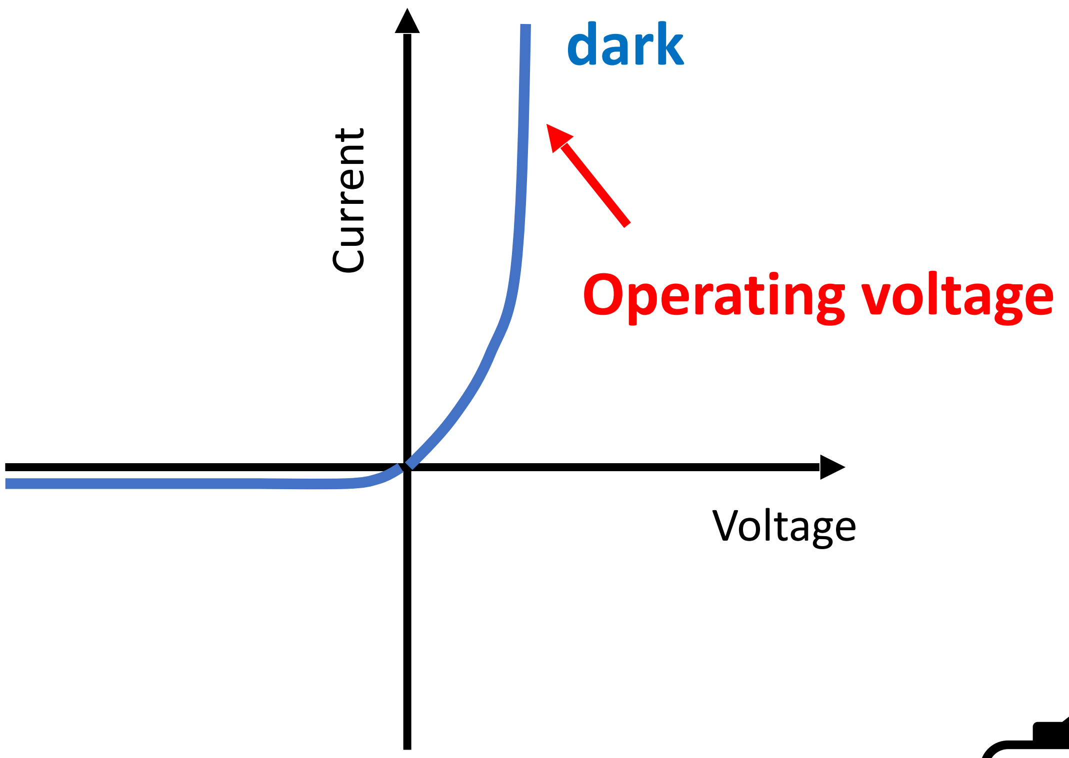
$$J_{total} = e\left(\sqrt{\frac{D_e}{\tau_e}}\frac{1}{N_A} + \sqrt{\frac{D_h}{\tau_h}}\frac{1}{N_D}\right)N_CN_Vexp\left(-\frac{E_g}{k_BT}\right)\left\{exp\left(\frac{eV_a}{k_BT}\right) - 1\right\}$$

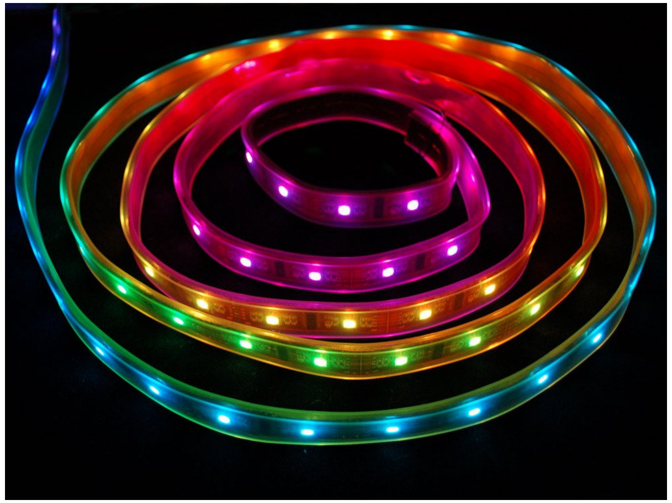
$$\approx e\left(\sqrt{\frac{D_e}{\tau_e}}\frac{1}{N_A} + \sqrt{\frac{D_h}{\tau_h}}\frac{1}{N_D}\right)N_CN_Vexp\left(-\frac{E_g}{k_BT}\right)exp\left(\frac{eV_a}{k_BT}\right)$$

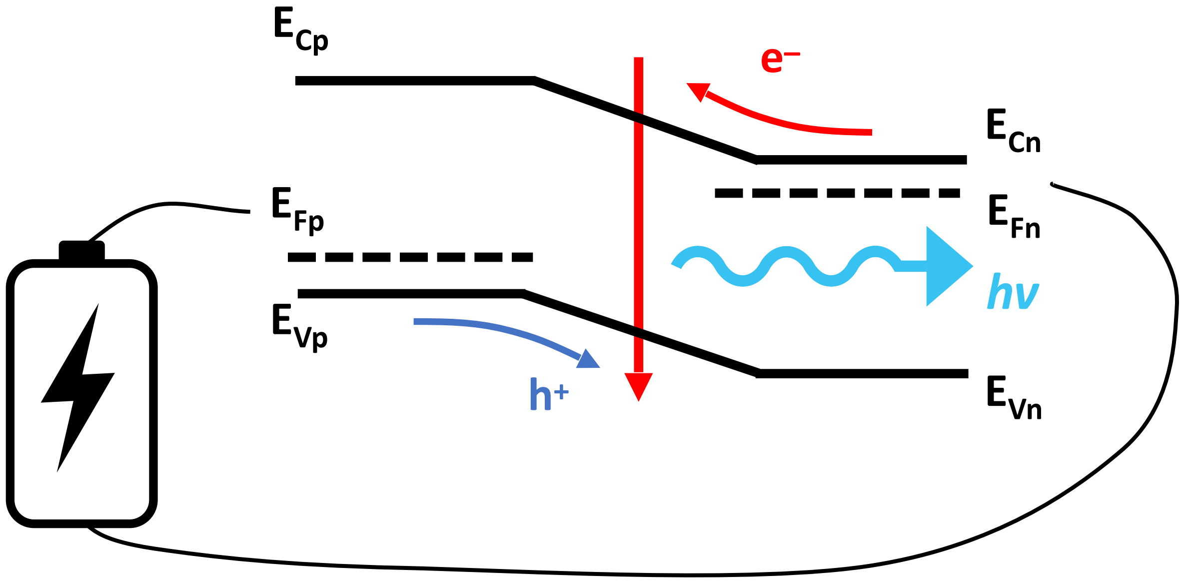
$$= e\left(\sqrt{\frac{D_e}{\tau_e}}\frac{1}{N_A} + \sqrt{\frac{D_h}{\tau_h}}\frac{1}{N_D}\right)N_CN_Vexp\left(\frac{eV_a - E_g}{k_BT}\right)$$



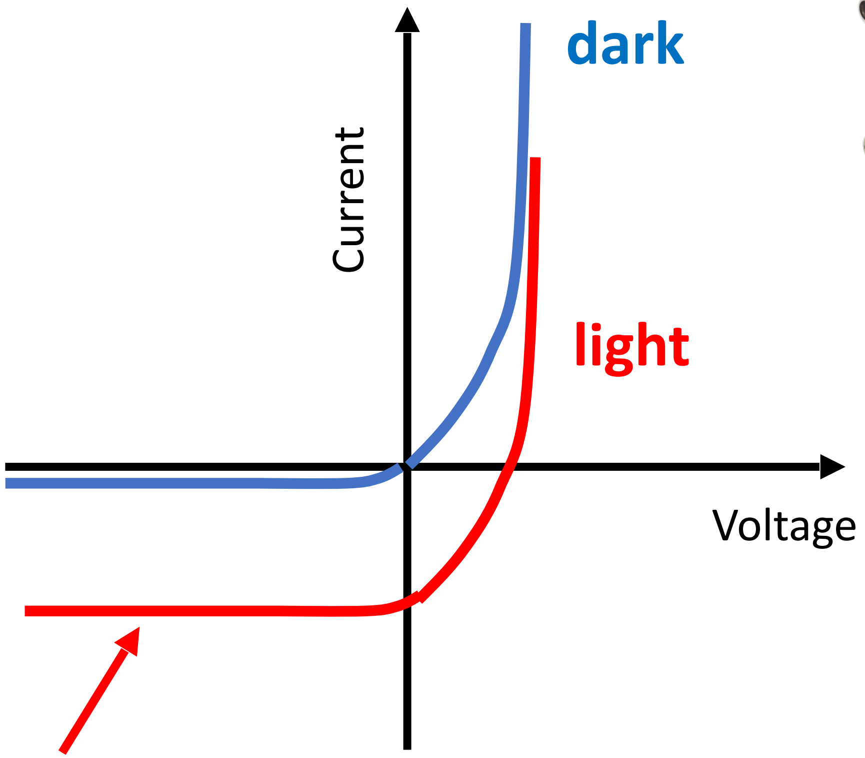
Illuminating pn Junction: LEDs and Laser Diodes

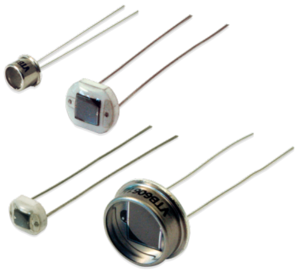


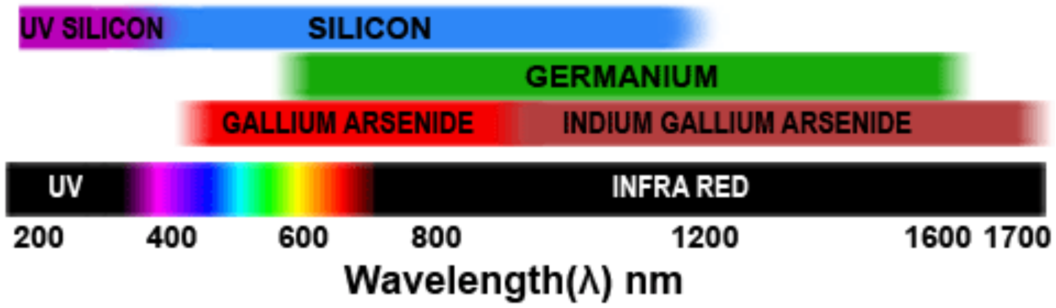


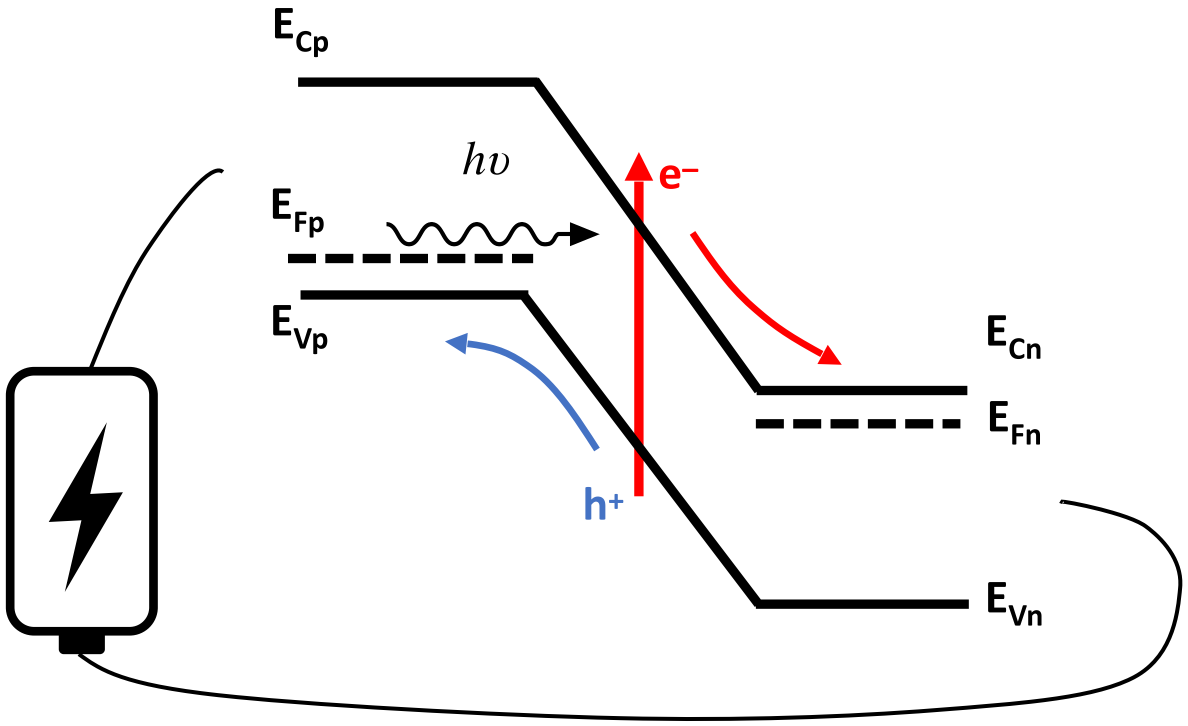


Photodiodes: Measuring Light Intensity









Operation voltage to increase E-field and collect as much current as possible and to improve speed.

Illuminated pn Junction: Solar Cells